STATISTICAL MANAGEMENT AGAINST EXPERIMENTAL WORK PLANNING

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Abstract: The mathematical theories concerning the plan of experimentation are relatively new. These were born in the 70s, and their rapid development and application is being encouraged by the intensive use of computers. Early statistical methods (regression analysis, statistical correlation, etc.), which were mainly used for mathematical modeling, supported the processing of results from the "passive experiment". During this, the mathematical methods of processing these results (collected data), are put into service in the last stage of an important process towards the formation of the mathematical model for the object under consideration. The notion of experiment plan which is used here, means laying out experiments based on a scheme previously designed in the form of the respective matrix. This means that specific mathematical methods must be used at all stages of the experiment. This can be decomposed in advance into the following notions: during the analysis of the relevant notes; before and during the experiment planning; during the processing of the results arising from the experiment; in the research strategy selection phase; in the final stage for the correct interpretation of data and results.
The classical approach to conducting an active experiment, when only one factor (variable) changed, while the others remained at the initial level, is practically not used in contemporary industrial facilities, because this method does not take into account the interactions of factors. Also, in this case a large number of experiments would have to be performed, but the reliability of the results would be at a low level. The cost of experimental research, according to current knowledge, is proportional to the square of the number of experts participating in them, while their efficiency is proportional to the square root of the total number of experiments. This means that experimentation is required to be optimized, because it does not make sense to make a very large number of experiments, the value of which would exceed the value of the positive effects that will result from their results. By optimizing the experiment it is meant to choose the most suitable mathematical model, but which requires a minimum number of experiments. It is rightly assumed that the active plan of experiments represents a possible optimal plan.
Keywords: Active experiment plan, Optimal model, Statistical analysis, Management of obtained experimental results

1. INTRODUCTION

In the active plan of the experiment all the factors that determine the technological process, change at once in harmony with the rules of the plan, while the results of the experiment, after proper processing enable the formation of one or more mathematical relations, which relations can be used. later for calculating the value of the independent variable. We call these mathematical relations obtained in this way a mathematical model of the object under consideration.
We generally distinguish these stages of the experimentation plan:
a) Collection and analysis of preliminary information.
b) Determination of dependent and independent variables, as well as the range of change of their values.
c) Determining the intended mathematical model.
d) Determining the plan of the experiment to be used.
e) Determining the method of data analysis.
f) Realization of experiments.
g) Verification of statistical hypotheses on the obtained results.
h) Proper processing of results from experiments.
i) Interpretation and presentation of results.
j) Appropriate recommendations regarding the application of the model used.
All of these actions are important and should not be neglected or underestimated.
Also, for each of these actions, some rules are defined, which the experimenter must adhere to.
In addition to these global rules, the developed intuition of the experimenter to "investigate" the results, as well as his professional practice are prerequisites, which guarantee the correctness of the obtained results (mathematical model).

2. DEFINING VARIABLES IN THE SYSTEM

At the beginning of the process review, it is required that all factors be determined with engineering precision, as well as, with objective methods, to assess the degree of impact of each factor in the process. If, for example, one of the influential factors is not taken into account, this will increase the error expressed in the results obtained. On the other hand, the inclusion of all process factors in the experiment program, significantly affects the increase of the degree of mathematical complexity of the task set, as well as the overall cost of the experiment.
In this regard, the experience and intuition of the experimenter should play a decisive role. With long experience related to any technological process, the necessary knowledge can be gained that enables the correct evaluation of those factors which have a decisive influence on the dependent variable which is elaborated.
Thus, it is required to identify the decisive factors as well as to determine the qualitative and quantitative properties.
In the theory of the experimental plan, several theoretical methods have been developed on the basis of which influential factors (independent variables) can be determined. Among these methods, to date, the so-called equilibrium method has gained practical verification, but the practice and intuition of the experimenter still remains the most accepted method for this issue.
However, when considering the degree of influence of various factors on the dependent variable, some basic rules should be used:
- That the range of change of variables is within the limits which from the technological point of view are "reasonable".
- That the variables do not take values that would cause complete disruption of the process.

From the beginning, too, the dependent variable (or variables) must be defined, which represents the system's response to the action of the factors.
This variable is taken from the purpose and features of the research, so it can be of technological nature (yield, quality, extraction rate, required granulation, etc.), or of economic nature (minimum total costs, maximum profit, cost of usage, etc.).
The dependent variable must possess a certain degree of sensitivity to changing factors, it must have the ability to measure and calculate easily, the ability to express itself in numerical form, have full physical meaning and never possess dualism.
From this it is obvious that the definition of independent and dependent variables, as well as the range of their change, can decisively affect the suitability of the acquired model for practical use. Unfortunately, their definition is not yet formatted, so in this research step, we mainly have to rely on the experience and intuition of the experimenter.

3. MODEL STRUCTURE AND PLAN

Contemporary engineering in the role of researcher can choose that model structure, which can best fit the real object it is examining. The concept of "best model" means that it offers better answers, gives satisfactory results in determining the future flow, as well as presents the object as reliably as possible in industrial conditions. To achieve this goal, it is recommended that at the beginning of the experiment, for the same object, different models be considered.
Usually the simplest model is considered first, which analyzes the linear effects:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i \qquad (1).$$

Then, the effects of the interaction of $x_i x_j$ factors are examined, for $i \neq j$:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i \neq j}^{n} a_{ij} x_i x_j \qquad (2).$$

Finally (in most cases) the action of the squares of the factors x_i^2 is examined, according to:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i \neq j}^{n} a_{ij} x_i x_j + \sum_{1}^{n} a_{ij} x_i^2 + \cdots \qquad (3).$$

Here is:

a - estimates of free polynomial coefficients;

y- (measured) system response;

x_{ij} - the independent variables in the system, or the values with which the experiments are performed.
Active experiment plan means the realization of experiments based entirely on the chosen plan. The chosen plan of the experiment determines in advance the values of the independent variables with which each experiment must be performed, ie determines in advance the conditions for conducting the experiment (or the location of the experiment).
Usually the plan of the experiment is laid out in the form of a plan matrix, where each row of it defines the conditions of the experiment, and each column-values that must take the independent variables in each experiment.
Let be the model (2) in the form of matrix equation. In this case it can be written:

$$Y = AX + E \quad (4),$$

where is:
Y- vector -column of expected values of the independent variable;
X- plan material;
A-vector-column of free coefficients.
The classical approach of determining A, using the method of smaller squares, thus laying the condition of minimization of E, results in the equation:

$$A = (X^T X)^{-1} X^T Y \quad (5).$$

The matrix (X^T X) is called the information matrix for the model under consideration.

4. STATISTICAL ANALYSIS OF A CONCRETE CASE

Further, a concrete case will be analyzed, where the values are given according to the following table:

Experiment (j)	**INPUTS**		**p_j**
	x_{1j}	**x_{2j}**	
1	12.7	6.7	5.8
2	7.7	10.7	7.8
3	5.7	5.4	4.8

Based on the form of the first degree polynomial we can write:

$$\begin{aligned} a_0 + 12.7a_1 + 6.7a_2 &= 5.8 \\ a_0 + 7.7a_1 + 10.7a_2 &= 7.8 \\ a_0 + 5.7a_1 + 5.4a_2 &= 4.8 \end{aligned} \quad (6).$$

The solutions of this system of equations are:

$$a_0 = 1.595 \quad ; \quad a_1 = 0.041 \quad ; \quad a_2 = 0.551 \quad (7).$$

Thus, the form of the approximate polynomial is:

$$p = 1.595 + 0.041x_1 + 0.551x_2 \quad (8).$$

Figure 1 graphically presents the functional dependence x_2 = f (p, x_1) for the different values of the parameter p. Analyzing the diagram it can be concluded:
- as the value of the variable x_1 increases, the value of the variable x_2 decreases;
- for the same value of the variable x_1, the value of the variable x_2 is greater for the greater value of the parameter p.

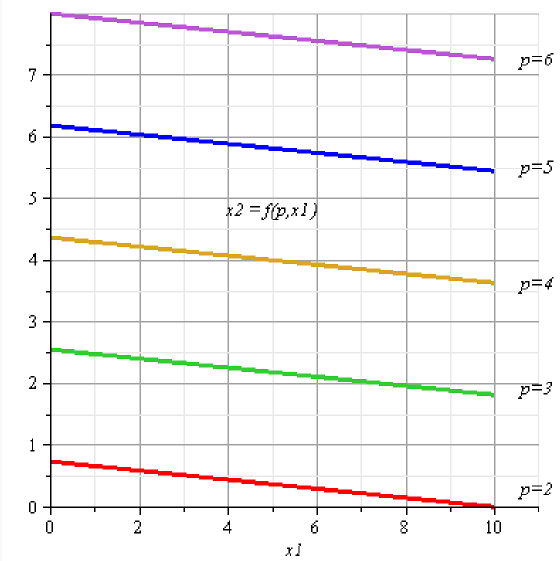

Figure 1: Functional dependence $x_1 = f\ (p, x_2)$ for the given approximate dependence.

One of the important quantities related to the management of statistical analysis has to do with determining the coefficient of elasticity.

The coefficient of elasticity of the dependent variable Y to the independent variable X is defined according to:

$$\varepsilon(Y,X) = \frac{\partial Y}{\partial X}\frac{X}{Y} \qquad (9),$$

so it is valid:

$$\varepsilon(X,Y) = \frac{\partial X}{\partial Y}\frac{Y}{X}$$
$$\varepsilon(Y,X) + \varepsilon(X,Y) = 1 \qquad (10).$$

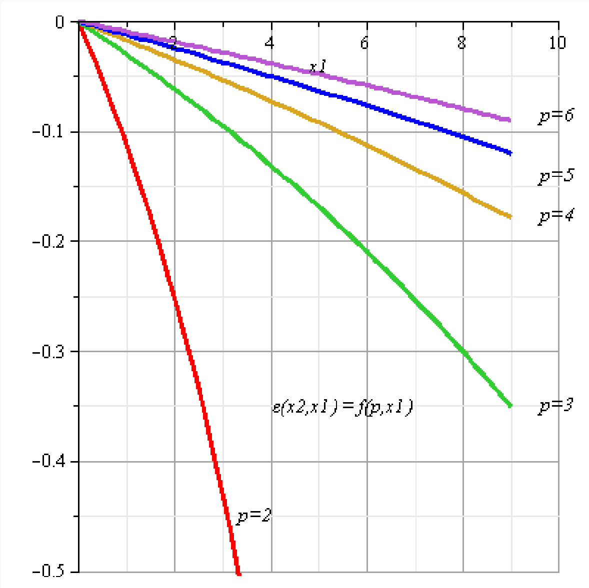

Figure 2: Elasticity coefficient for the problem analyzed.

Analyzing the diagram according to figure 2, it can be concluded:

- as the variable x_1 increases, the coefficient of elasticity decreases with respect to the value, so that the downward trend is more pronounced for the smallest parameter p;

- for the same value of the variable x_1 the coefficient of elasticity is smaller for the parametric curve with the value of the parameter p smaller.

For example, for $x_1 = 3$ and $p = 3$ is $\varepsilon (x_2, x_1) = - 0.1$, which means that when x_1 increases by 1%, the variable x_2 decreases (minus sign) by 0.1%, and so on.

REFERENCES

Avetisov, S. E., Novikov, I. A., Bubnova, I. A., Antonov, A. A., & Siplivyi, V. I. (2010). Determination of corneal elasticity coefficient using the ORA database. Journal of refractive surgery, 26(7), 520-524.

Bellman, E.R., & Dreyfus, E.S. (2013). Applied Dynamic Programming, Princeton University Press, Princeton, New Jersey

Churchman, C.W., Ackoff , R.I., Arnoff, E.L. (2017). Introduction to Operations Research, John Wiilley and Sons, New York

Harvey, W.M. (2018). Principles of Operations Research, Prentice-Hall, New Jersey

Kocherlakota, N. R. (1990). Disentangling the coefficient of relative risk aversion from the elasticity of intertemporal substitution: An irrelevance result. The Journal of Finance, 45(1), 175-190.

Manson, N. J. (2006). Is operations research really research?. Orion, 22(2), 155-180.

Stoller, D.S. (2017). Operations Research, Process and Strategy, University of California Press, Los Angeles

Vajda, S. (2013). Mathematical Programming, Addison – Wesley Publishing Company, Inc. London